Injecting Periodicities: Sieves as Timbres

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Abstract — **Although Xenakis's article on sieves was published in 1990, the first extended reference to Sieve Theory is found in the final section of 'Towards a Metamusic' of 1967. There are certain differences among the two. The section of 'Metamusic' is titled 'Sieve Theory' whereas the 1990 article simply 'Sieves'. The latter is more practical, as it provides a tool for treating sieves (with the two computer programmes). These two writings mark two periods: during the first, sieves were of the periodic asymmetric type, with significantly large periods; in the more recent, they were irregular with absent periodicity. Also, the option of a simplified formula only appears in the 1990 article: there is a progression from the decomposed formula to the simplified one. This progression reflects Xenakis's aesthetic of sieves as timbres and is here explored under the light of the idea of injected periodicities.**

KEYWORDS Xenakis, Sieve Theory, Analysis

I. Canonical Form

A sieve involves the combination of two or more *modules*. A module is notated by an ordered pair (m, r) that indicates a *modulus* (period) and a *residue* (an integer between zero and m-1) within that modulus. Given that there might be several equivalent formulae for a given sieve, we can choose among several alternative decompositions of the moduli into two or more factors. This redundancy of formulae is overcome by prime factorisation, as this is aimed at rendering a decomposed form of a composite number. According to the Unique Factorisation Theorem any integer $\alpha > 1$, can be uniquely written as

$$\alpha = p_1^{\,l} \cdot p_2^{\,m} \cdot \ldots \cdot p_k^{\,n}, \quad (1)$$

where $p_1, p_2, \ldots, p_k$ prime numbers, $p_1 < p_2 < \ldots < p_k$ and $l, m, n \in \mathbb{N}^*$; this is called the *canonical form* of α. It is easily inferred from the theorem, that since two numbers α, β, are coprime then α^m, β^n (where $\alpha, \beta, m, n \in \mathbb{N}^*$) are coprime as well. Thus co-primality of the factors is secured.

The canonical form of 12 is $2^2{\cdot}3$. When these factors correspond to moduli, the literal intersection would have to be $(2, r_1){\cdot}(2, r_2){\cdot}(3, r_3)$. It is obvious that $(2, r_1){\cdot}(2, r_2)$ is not a valid option. In the case that $r_1 = r_2$ (either as such or after modular reduction) the intersection implies that $(2, r_1) = (2, r_2)$. In the case they are different the intersection does not exist. (After modular reduction of the residues, there can be no intersection of modules that share the same modulus.) We should therefore resolve any exponentials before treating prime factors as the elementary moduli of a period: $12 = 2^2{\cdot}3 = 4{\cdot}3$. Therefore, $(12, r) = (4, r_1){\cdot}(3, r_2)$.

II. Types of Formulae

A. Decomposed Formula

The decomposed formula is the one that employs elementary moduli that derive from the canonical form of the sieve's period. As I have shown, of this type are the formulae that use moduli 4 and 3 to express a sieve whose period is 12. The combination of two or more moduli does not necessarily suggest a decomposed formula. In general, a decomposed formula is the one that only employs moduli that are either primes or derive from a power of a prime as found in the canonical form. Thus, an intersection that involves moduli 4 and 6 is not part of a decomposed formula, whereas one that involves 4 and 3 is.

B. Simplified Formula

A simplified formula consists of unions of single modules. It might not always represent a sieve according to a single modulus that corresponds to the period. As I will show later on, there are two alternative simplified formulae that are based on two different levels of periodicity. Starting with a formula that possibly contains all three logical operations, we can transcribe it into a series of unions of intersections and reduce each intersection to a single module. When several elementary moduli are employed, the decomposed formula might not offer a synoptic and rigorous view of the sieve's structure, and a simplified one might be a preferable option.

III. Programme

A. Generation of Points

The intention to decompose a modulus into its canonical form is also evident in the computer programme Xenakis developed for the 'generation of points on a straight line from the logical formula of the sieve'. At the fourth stage the programme asks the user whether it should decompose the modulus of each module into prime moduli. This might seem to be superfluous in the course of the programme. Decomposition is part of a process that simplifies and then decomposes the moduli, only in order to display again the simplified formula of the sieve as unions of single modules. This does not affect the sieve in any manner and seems to be there only to provide the user with two alternative formulae. The behaviour of the programme is interesting when it is given a formula that includes moduli that do not derive from the canonical form: the programme reduces the intersections of two or more modules into one and then provides a decomposition into prime moduli. For example, if we input an intersection that involves moduli 4, 5 and 6, then the programme suggests a reduction of these moduli to a single module with modulus 60 (the Lowest Common Multiple). Afterwards, it suggests a 'decomposition into prime moduli' 4, 3 and 5, and finally displays the simplified notation again, reduced to a single module with modulus 60 (the product of 4, 3 and 5). This is because 4, 5, and 6 cannot be derived from the canonical form of any number.[1]

At a first glance, it might seem strange that decomposition into prime moduli includes modulus 4, which is not a prime. Xenakis provides a demonstration of the programme (as well as of the inverse one, discussed below). I remind that it was first published in the 1990 article and then in the 1992 edition of *Formalized Music*. In the former edition the programme's prompts appear in French and in the latter in English. However, this is not the only difference; in the French edition we read: 'decomposition into coprime modules?',[2] whereas in the English: 'decomposition into prime modules?'. Although the two expressions appear to be inconsistent, they are both correct. If we prompt the programme to decompose modulus 12 the result involves moduli 4 and 3. At first, the French expression about co-primality seems to be true: although 4 is not a prime, the two moduli are coprime. In this sense the English expression is not valid. But the two versions seem to refer to different sub-stages of the process. The French expression is true only after having resolved the powers of the primes as found in the canonical form.

On the other hand, I have demonstrated that co-primality is not achieved through a free selection of numbers, but is arrived at through the canonical form of the modulus. The canonical form of a number, refers to the *order* prime factors appear. It is not merely an unordered collection of its prime factors, but these are put in the order found in (1). This is the reason why the programme's output at this stage is $4{\cdot}3$ and not $3{\cdot}4$ ($12 = 2^2{\cdot}3 = 4{\cdot}3$). Therefore, the English expression refers to the stage before the actual output, which is no other than the canonical form (before the resolution of the exponentials). So, in the latter expression we should read $12 = 2{\cdot}2{\cdot}3$, the three prime factors of 12. Consequently, the (more recent) English expression is also true. This stage (the stage of the canonical form) is neither explicit in the programme, nor in the theory as demonstrated in the article. It remains a hidden element, but implied both in the former and the latter.

B. *Generation of the Logical Formula*

Gibson has demonstrated that this programme is based on a symmetric conception of sieves (see [2]: 55). The algorithm ignores the actual period and suggests a theoretical one; in the case of the major scale it does not reach a correct estimation of the octave, unless the input is a minimum of octaves. However, this minimum cannot be established, as it depends on the density of the given sieve.

The problem with constructing sieve formulae is related to the determination of the period: without knowing the period we are unable to analyse the internal symmetry of a sieve. This practical analytical problem lies even before the one caused by the redundancy of formulae for a given sieve. Determining the periodicity therefore, seems to be located at the foundations of sieve analysis, in the sense that without knowledge of the external aspect of a sieve it seems impossible to define any of its other aspects. One could suggest treating a given collection of pitches as the occurrence of a single period, but this periodicity would still be a hypothetical one.

Although the knowledge of the period is necessary to decompose a sieve, Xenakis clearly ignores it. I remind that periodicity is for Xenakis an 'identity in time' ([10]: 268), while he situates sieves in the realm of outside-time musical structures. In this sense, it is not surprising that his algorithm treats sieves analysing directly their internal structure. In fact, from the late 1970s to the early 1990s Xenakis progressed towards extremely irregular sieves. As himself said: '[a scale] can be dispersed over the whole range of pitches' ([11]: 15).

IV. Non-Periodic Sieves

Even in the sieves of the 1960s the internal structure was more important: highly complex and asymmetric. Sieve Theory was intended to reveal internal symmetries, hidden under the surface. It seems therefore that the overall periodicity had never been Xenakis's central concern. The formula that the programme derives from a given sieve is of the simplified type; it immediately searches for internal symmetries in the form of periodicities as found in the intervallic structure of the sieve.

The turning point of Xenakis's evolution in sieve-based composition is marked by *Jonchaies* (1977). In the preface to the score he clearly states that the work 'deals with pitch "sieves" (scales) in a new way' [7]. The sieve of this work is asymmetric with a prime period, thus excluding a decomposed formula. It is the first time that such a type is used and this clearly verifies Xenakis's tendency to notate sieves with a simplified formula. In terms of how the sieve of *Jonchaies* is treated in the composition, there is a particular technique which Makis Solomos has termed 'halo sonore' (see [4]: 84). This is important to mention here as it reveals the shift to a new aesthetics of sieves. Xenakis himself had not commented extensively on this technique but only described it in the preface to the score of a work from the same period, *Nekuia* (1981), as 'multiplicities of shifted melodic patterns, like in a kind of artificial reverberation' [8]. The result is a kind of heterophony, where the outcome is not of any traditional type of treatment of pitch such as melody, polyphony, etc; not even the type of the set-

[1] I am using the corrected version of the programmes, as provided in [5: 291-303]. For other works on the implementational character of Sieve Theory see [1] and [3].

[2] '[D]écomposition en modules premiers entre eux ?' ([9]: 69). According the terminology I am using, the word 'modules' (both in the French and the English versions) should read as 'moduli'.

theoretical treatment in his symbolic music (*Herma*); indeed, in the late music of Xenakis sieves become timbres rather than pitch sets or scales.

In the sieves following *Jonchaies* Xenakis avoids periodicity and therefore, the external aspect of the sieve is removed. Furthermore, these sieves share a certain aesthetic: they are characterised by an irregular distribution of intervals which are dispersed over the whole range. The size of these intervals is contained between a semitone and a major 3rd (M3rd). The selection of these intervals is related to an aesthetic criterion that seems to have influenced most of Xenakis's recent output: the construction of sieves is inspired by the Javanese *pelog* with its interlocking 4^{ths} (hence the characteristic interval succession 1 4 1 semitones). Asymmetry is taken to its extreme and a decomposed formula seems to be superfluous; a simplified formula is more appropriate to describe the sieve's internal structure.

V. Injected Periodicities

Before we compare the two types of formulae, we need to determine another level of periodicity, different than the overall period of the sieve. The two levels of periodicity correspond to the two different kinds of approach by the two computer programmes. In the first one, the period is known and a simplified formula is an alternative to the decomposed one; the second aims at representing the sieve's internal structure, without presupposing its overall period.

A. External Periodicity

The distinction between two levels of periodicity is not equivalent to the one of internal and external symmetry, as it does not consist a criterion for classifying sieve-types. It refers to the possibility of two alternative simplified formulae and reflects Xenakis's aesthetic of sieve-construction. The external periodicity of a sieve is no other than its overall period. A simplified formula based on the external periodicity, indicates the period by only including moduli that are congruent (or equal) to the period. I remind here that the external periodicity is also indicated by the decomposed formula; therefore, any simplified formula that derives from the decomposed one is based on the external periodicity.

B. Internal Periodicities

This level is found at the interior of the sieve where several simple periodicities take place. These internal periodicities are shown by the simplified formula of a sieve in the form of single modules. The second computer programme is aimed at suggesting a simplified formula in an way that describes the sieve's internal periodicities. The overall period is either not known or not taken into account. This is more appropriate for the non-periodic sieves from Xenakis's more recent output.

Although the sieve of *Jonchaies* is not of the type described here (i.e. it *is* periodic), it marks Xenakis's general approach to sieves as timbres. The inspiration for this orchestral work, the composer comments, comes from the results of his research in sound synthesis for *La légende d'Eer*. Especially towards the ending of *Jonchaies* there is a striking aesthetic resemblance with this electroacoustic composition of the same year (1977). In the preface to the score of *Jonchaies*, after mentioning the novelty in the treatment of sieves, Xenakis briefly describes his inspiration by his view on sound synthesis: 'one starts from noise and […] periodicities are injected to it' [7]. Admittedly, this is a possibility offered by stochastics; but the inspiration from electroacoustic to instrumental composition (and vice versa) can be seen in relation to Sieve Theory as well. The idea of individual periodicities is not extremely different from the original idea of stochastics: individual elements are distributed in such a way that they are not intended to be perceived as such, but to create a 'multitude of sounds, seen as a totality' ([10]: 9).

At the end of his article, Xenakis argues that the inverse, i.e. the application of Sieve Theory to sound synthesis is 'quite conceivable'. In that case the periodicities injected to noise would be the simple modules in a simplified formula. This is also related to Xenakis's general approach to sound synthesis, which does not necessarily imply electronic music; it is rather 'a general term referring to the production of artificial sound by *some means*' ([6]: 92; italics added). In this sense, it is more than conceivable, instead of starting from noise, to start from the total chromatic throughout the audible range and 'inject periodicities' in order to construct a sieve that produces a certain timbre. The analytical algorithm proposed by Xenakis is, in this sense, an effective method for analysing such timbres.

Whereas with the decomposed formula the problem of redundancy is solved using prime factorisation, the redundancy of simplified formulae is surpassed by the idea of injected periodicities. The algorithm suggested in Xenakis's article starts from the smallest possible modulus at the earliest possible starting point; the result is a unique formula that corresponds precisely to this idea.

C. From Hidden Symmetry to Hidden Periodicities

The absence of symmetry in a sieve, prompts one to study a more hidden symmetry. Likewise, the absence of periodicity can raise the question of hidden periodicities. In order to demonstrate the transition to non-periodic sieves, let us take for example the sieve of *Ata* (1987) as found in Xenakis's pre-compositional sketches (see Fig. 1 – numbers refer to semitones). It consists of 37 points and its range is six octaves and a minor 6th (m6th).

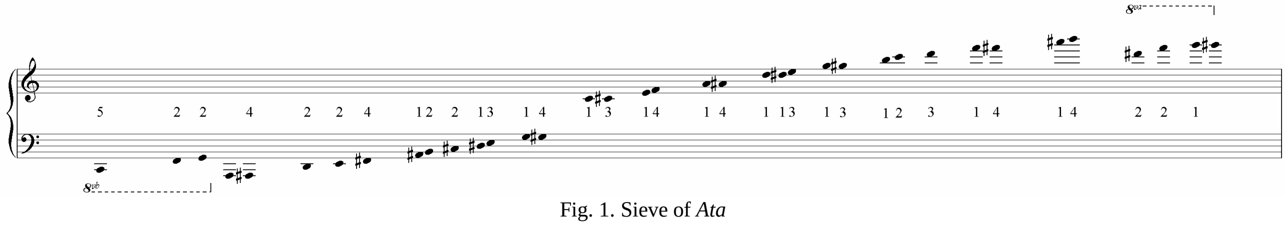

Fig. 1. Sieve of *Ata*

TABLE I.
DECOMPOSED MATRIX FOR THE SIEVE OF *ATA*

16_i

5_j	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	**0**	**65**	**50**	35	20	**5**	**70**	**55**	**40**	**25**	**10**	**75**	**60**	**45**	30	15
1	**16**	1	**66**	**51**	**36**	21	6	**71**	**56**	**41**	26	11	76	61	**46**	**31**
2	**32**	17	2	67	**52**	**37**	**22**	**7**	72	57	42	**27**	12	77	**62**	47
3	48	33	**18**	3	68	53	38	**23**	8	73	58	43	**28**	13	78	63
4	64	49	34	19	4	69	54	39	24	**9**	74	**59**	44	29	**14**	**79**

The interval structure is asymmetric and consists of intervals between a semitone and a M3^rd (with one exception of a perfect 4^th). The absence of periodicity indicates that a decomposed formula is not appropriate. In that case, a hypothetical formula would have to consider 80 (6·8^ve + m6^th) as the period and decompose it: $80 = 2^4 \cdot 5 = 16 \cdot 5$. Thus a 16 by 5 matrix, of the type that Gibson uses for periodic sieves (see [2]: 48 ff.), would render the sieve's structure. The matrix for the decomposed sieve of *Ata* is shown in Table I (0 = C1 and C4 = middle C). However, its periodicity is an arbitrary one and contradicts Xenakis's assertion that a scale should not be periodic nowhere throughout its range. What is of interest and not shown in the matrix of Table I, is the internal structure as a multiplicity of elementary, individual periodicities. This is shown by a matrix that corresponds to the simplified formula.

The simplified matrix for the sieve of *Ata* is shown in Table II. It is a generalisation of the demonstration of Xenakis's programme as shown in [2: 53]. The top row shows the consecutive number of the points. The intervallic structure of the sieve (in semitones) is shown in italics under the actual points of the sieve. The modules are under the label (*M*, *I*, *R*), which stands for: Modulus, Initial point, Reprises of the modulus. In the simplified matrix the residues are not reduced according to the modulus; they are simply considered as starting points (*I*). As the starting point of a module (i.e. of an internal periodicity) can be located anywhere in the sieve, it is kept as such, even if it is larger than the modulus. Thus, instead of substituting (5, 40) with (5, 0), it is more important to indicate that a periodicity of 5 semitones, or of a (perfect) 4^th is initiated at point 40, right at the middle of the sieve. This is one of

TABLE II.
SIMPLIFIED MATRIX FOR THE SIEVE OF *ATA*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	0	**5**	**7**	**9**	**10**	**14**	**16**	**18**	**22**	**23**	**25**	**27**	**28**	**31**	**32**	**36**	**37**	**40**	**41**
M, I, R	*5*	*2*	*2*	*1*	*4*	*2*	*2*	*4*	*1*	*2*	*2*	*1*	*3*	*1*	*4*	*1*	*3*	*1*	*4*
25,0,3	+										+								
18,5,4		+								+									+
24,7,3			+											+					
14,9,5				+						+							+		
15,10,4					+						+							+	
23,14,2						+											+		
25,16,2							+												+
19,18,3								+									+		
19,22,3									+										+
19,27,2												+							
17,28,3													+						
20,31,2														+					
19,32,2															+				
15,36,2																+			
5,40,8																		+	
14,52,2																			

	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
	45	**46**	**50**	**51**	**52**	**55**	**56**	**59**	**60**	**62**	**65**	**66**	**70**	**71**	**75**	**77**	**79**	**80**
	1	*4*	*1*	*1*	*3*	*1*	*3*	*1*	*2*	*3*	*1*	*4*	*1*	*4*	*2*	*2*	*1*	
25,0,3			+												+			
18,5,4								+								+		
24,7,3						+											+	
14,9,5				+							+						+	
15,10,4						+							+					
23,14,2									+									
25,16,2												+						
19,18,3							+								+			
19,22,3									+								+	
19,27,2		+									+							
17,28,3	+									+							+	
20,31,2				+										+				
19,32,2				+									+					
15,36,2				+								+						
5,40,8	+		+			+			+		+		+		+			+
14,52,2					+							+						+

the aspects of Xenakis's more practical approach to sieve-construction, that characterises his more recent output. The third entry in the brackets (*R*) shows the number of repetitions of each module. The matrix itself shows the number of points each module covers, which is always $R + 1$. In the matrix for the sieve of *Ata*, the greatest number of points covered by a single module belongs to (5, 40), which covers 9 points; this means that a periodicity of a 4th (5 semitones) is repeated 8 times; therefore we note (5, 40, 8).

The sieve's individual (internal) periodicities can only be considered if they repeat at least two times. In other words, a module must cover at least three points. Consequently, its modulus must be of a size less than half of the difference between the sieve's ambitus (range) and the module's starting point. If a is the ambitus of the sieve, then it must be valid that $M \leq (a - I)/2$ in order to secure internal periodicities.[3] In the sieve of *Ata* the ambitus is 80 semitones. If the starting point of a module is 0, then its modulus can only be equal to or smaller than 40 semitones. Likewise, a module starting on point 52 can only have a modulus up to $(80 - 52)/2 = 14$ semitones. This is found in the last entry in the matrix as module (14, 52). In these two cases, a modulus larger than 40 or 14 semitones respectively, cannot be considered as an internal periodicity of the sieve. Indeed, the sieve of *Ata* is shown to be carefully constructed to only include modules that repeat themselves at least two times.

In the sieve there are16 modules that produce 37 points. These modules are classified according to their size and are shown in Table III. The leftmost column shows the canonical form of each modulus, and the rightmost column the musical interval each modulus represents. The table offers synoptic information on the periodic intervals that make up the sieve. As I already mentioned, the greatest number of repetitions of a single module belongs to the interval of the 4th. Another characteristic of the sieve is that there are no moduli of octaves, except perhaps of the three double octaves shown in the table. If we add all the repetitions of a modulus found in more than one modules, we see that the interval of an octave plus a 5th is found ten times in the sieve (see the four modules that share modulus 19 in Table III). However, this does not suggest a more decisive contribution of the interval 8ve + 5th to the sieve's structure than of the 4th, since it is not a case of successive repetitions of an interval. Such information is valuable only in order to give a more general character of the sieve's periodic intervals. Moreover, the repetition of the 4th eight times is a much more perceivable characteristic also because of its relatively small size. The larger interval in the successive intervallic structure of the sieve is the M3rd (with one exception) and therefore the 4th is the smallest interval one can expect to find as a modulus (in the sense that a module of a M3rd would confine the sieve's intervallic structure to intervals of m3rd or smaller).

[3] The reverse is not necessarily true: a modulus $M \leq (a - I)/2$ might not in all cases repeat itself at least two times. This is simply because the module might not always meet points in the sieve (in which case of course it is discarded).

TABLE III.
THE MODULES IN THE SIMPLIFIED FORMULA FOR THE SIEVE OF *ATA*

Canonical Form	*M, I, R*	*Interval*
5	5, 40, 8	4th
2·7	14, 9, 5 14, 52, 2	8ve + M2nd
3·5	15, 10, 4 15, 36, 2	8ve + m3rd
17	17, 28, 3	8ve + 4th
$2 \cdot 3^2$	18, 5, 4	8ve + tritone
19	19, 18, 3 19, 22, 3 19, 27, 2 19, 32, 2	8ve + 5th
$2^2 \cdot 5$	20, 31, 2	8ve + m6th
23	23, 14, 2	8ve + M7th
$2^3 \cdot 3$	24, 7, 3	2·8ve
5^2	25, 0, 3 25, 16, 2	2·8ve + semitone

D. *Decomposition of Internal Periodicities*

Xenakis put forward the idea of the decomposition of a modulus as a method to compare different sieves: in particular, we can study their degree of difference and define a notion of distance between them (see [10]: 270). Any individual periodicity might have a composite modulus, which can be decomposed in order to allow comparison between all periodicities in the intervallic structure of a sieve. In this way we can render the sieve's more 'hidden' periodicities.

A notion of distance between two moduli can be defined through their decomposition. Thus we can study the structure of a single sieve before comparing different ones. In the sieve of *Ata* the interval of the 4th has been shown to be the most characteristic. One of the reasons for this is that it is the smallest modulus in the simplified formula. A notion of distance can be created between congruent moduli. In order to do this we need to define a unit distance. This unit can be the Greatest Common Divisor (GCD) of two or more moduli. For example, for numbers 12 and 18, 6 can be defined as the unit distance; the two numbers are 1 unit apart. This can be also useful in grouping more than two moduli in a formula.

The interval of the 4th is a prime number (5) and can be found as a constituent element of other moduli in the sieve. These are 15, 20, and 25. Therefore, four moduli out of ten (there are ten moduli and sixteen modules) are congruent modulo 5. These are the intervals of 4th, 8ve + m3rd, 8ve + m6th, and 2·8ve + semitone. If the unit distance of these moduli is the 4th, their size can be thought of in terms of how many 4ths they contain. Therefore, the above intervals can be written as follows: 4th, 3·4th, 4·4th, and 5·4th. Table IV and Fig. 2 indicate all the four moduli that are congruent modulo 5, the 4th as found in module (5, 40). In Fig. 2 the 4th is shown on a first level above the pitches; below the pitches the 8ve + m3rd; on a second level above the 4th, the 8ve + m6th; and on the top level the 2·8ve + semitone. The four moduli are found in 6 modules

TABLE IV.
MODULI CONGRUENT MODULO THE PERFECT 4^{TH} IN THE SIEVE OF *ATA*

Modulus	*Interval*	*Canonical Form*	*Multiples of GCD*
5	4^{th}	5	$1·4^{th}$
15	8^{ve} + $m3^{rd}$	3·5	$3·4^{th}$
20	8^{ve} + $m6^{th}$	$2^2·5$	$4·4^{th}$
25	$2·8^{ve}$ + semitone	5^2	$5·4^{th}$

in total: (5, 40), (15, 10), (15, 36), (20, 31), (25, 0), and (25, 16). These six modules produce eighteen points of the sieve. In particular, the modules congruent modulo 5 are more than the one third of the modules in total (sixteen modules), and produce almost half of the sieve's points (eighteen out of thirty-seven). Another interesting observation is that from these eighteen points, eight are produced by the 4^{th} itself. The importance of this interval is found elsewhere in Xenakis's music and comments. As analysis has shown, it is found in the inside-time treatment of sieves as well.

The grouping of moduli can be carried out for other intervals taken as the unit distance. For example, there is another combination of four moduli that are congruent to the interval of the tone. These are moduli 14, 18, 20, and 24, which correspond to the following intervals: 8^{ve} + 2^{nd}, 8^{ve} + tritone, 8^{ve} + $m6^{th}$, and $2·8^{ve}$. In terms of the $M2^{nd}$ as a unit distance, these intervals can be written as $7·2^{nd}$, $9·2^{nd}$, $10·2^{nd}$, and $12·2^{nd}$. However, the interval of the 2^{nd} does not appear in the sieve as such. If it appeared as part of a module it would reduce the intervals for some part of the sieve's intervallic structure to a succession of semitones and tones. The same grouping of moduli can be also carried out for the interval of the $m3^{rd}$. From the three options of the tone, the $m3^{rd}$, and the 4^{th}, obviously the latter is preferable. This is due to two reasons: firstly, among the three moduli, the 4^{th} is present as a periodic interval extending for half of the sieve's range, and secondly, in the sieves of Xenakis's more recent output the intervallic structure contains intervals up to a $M3^{rd}$; therefore the 4^{th} is the smallest interval one can expect to find as a modulus.

When one analyses a sieve, one might expect to also find moduli that are prime numbers. In our example, these are moduli 17 (8^{ve} + 4^{th}), 19 (8^{ve} + 5^{th}), and 23 (8^{ve} + $M7^{th}$). As a general characteristic, when several elementary moduli (internal periodicities) produce a sieve, these are expected to be coprime; the unit distance between coprime moduli is the semitone (their GCD). The attempt to define a unit distance refers to an attempt to group moduli according to a unit larger than one. But such a procedure is not irrelevant from decomposing a modulus into its constituent elements. Therefore, a unit must not be decomposable itself. As in the Sieve of Eratosthenes, the possible intervals for the unit distance must be sought among the primes. If for example, there was a module covering a large range of the sieve with modulus 6 semitones (a tritone), then this would have to be decomposed to 2·3, and then decide which of the two intervals (the tone or the $m3^{rd}$) would be the unit distance. This of course, would raise questions relating to the validity of a unit distance that is not present on the sieve's structure. Xenakis referred to 'hidden symmetry' which in the more recent sieves became 'hidden periodicities'; in this sense a unit distance that is not present can be thought of as a deeper level of periodicity. On the other hand, Xenakis analysed his sieves according the appearance of the modules as they were given by his algorithm. This description of a sieve depends upon general characteristics such as the proportion between the number of modules and the number of points produced, the number of repetitions of a module, etc. The degree of difference is enabled by a definition of a unit distance according to the decomposed moduli. In this sense, coprime moduli are considered to be the most different. Consequently, grouping modules refers to groups of non-coprime moduli. The more the non-coprime moduli that share an GCD, the more homogeneous the sieve. In general, when all moduli of the sieve are non-coprime then they are all divided by their GCD in order to render their degree of difference.

Decomposing a sieve into its constituent elements is a quantitative method in order to study it in terms of its difference *in degree* from other sieves. Xenakis's aesthetic of sieve construction has been influenced by two factors: (a) the pelog scale, and (b) his view on sound synthesis. This resulted into a general type of sieve that is different *in kind* from the sieves of the 1960s. Therefore, the study of the degree of difference among sieves must be aimed at comparing sieves of the same kind. The recent output of Xenakis abounds in sieves of the same kind, often thought of in terms of versions of original sieves. The general characteristics described here are also to be taken into account in the analysis of sieves. These characteristics reflect the two aesthetic factors mentioned: (a) the intervallic structure consists of intervals between a semitone and a $M3^{rd}$, and (b) sieves are constructed (and analysed) according to the idea of injected periodicities.

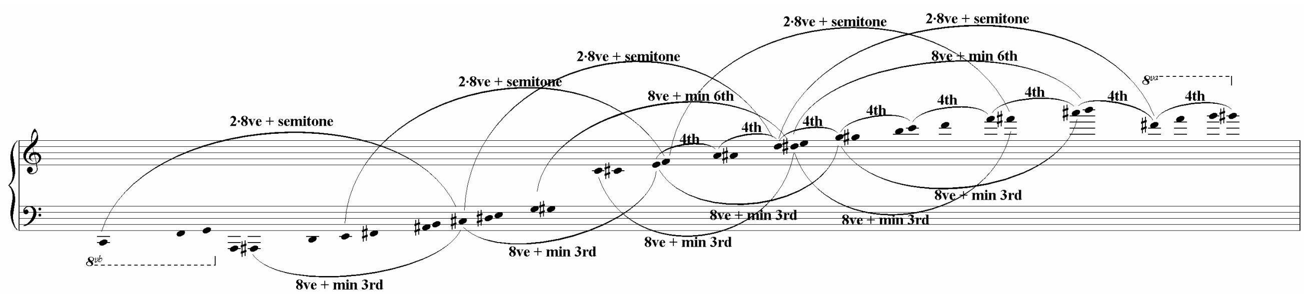

Fig. 2. Sieve of *Ata* and its Moduli Congruent Modulo the Perfect 4^{th}

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